B.Sc. Semester-II Examination, 2023

MATHEMATICS [Programme]

Course ID : 22118 **Course Code : SP/MTH/201/C-1B**

Course Title : Real Analysis

[OLD SYLLABUS]

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

a) Define limit points of a set. What are the limit points of $\mathbb{Q}$, the set of rational numbers?

b) Prove or disprove that for every real number x, there exists a positive integer n such that $n > x$.

c) Determine whether the sequence $\{x_n\} = 2^{1/n}$ converges or diverges, and if converges find its limit.

d) Justify whether the set of integers $\mathbb{Z}$ is open in $\mathbb{R}$.

e) Prove that $\sum \frac{1}{(n+1)(n+2)} = 1$.

f) Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

g) Test the convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

h) Check whether the sequence $\left\{\frac{2n+3}{3n+4}\right\}$ is bounded or not.

UNIT-II

2. Answer any **four** from the following questions: 5×4=20

a) State the Density theorem for rationals in R. Using this theorem, prove that between any two real numbers there is always an irrational number.

b) Define an enumerable set. Show that the union of two enumerable sets is enumerable.

c) Show that the series is conditionally convergent

$$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{n+1}.$$

Weather Dirichlet's test is applicable for this? Justify. 4+1

d) Find sup A and inf A, where

$$A = \left\{\frac{n+(-1)^n}{n} : n \in \mathbb{N}\right\}.$$

e) Find $limsup\{x_n\}$ and $liminf\{x_n\}$ where $\{x_n\} = (-1)^n(1+\frac{1}{n})$.

f) i) Verify Bolzano-Weierstrass theorem for the set $S \subset \mathbb{R}$ where $S = \{\frac{n-1}{n+1} : n \in \mathbb{N}\}$.

ii) Give an example of infinite set $S \subset \mathbb{R}$ such that S is a proper subset of the derived set S'. 4+1

UNIT-III

3. Answer any **one** of the following questions: 10×1=10

a) i) Prove that an absolutely convergent series is always convergent. Hence, prove that $\sum \frac{\sin n\theta}{n^2}$ is a convergent series.

ii) Find the limit of the sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n$.

b) i) Prove that the sequence $\{u_n\}$ defined by $u_n = \sqrt{3u_n}, u_1 = 3$ is monotonically increasing and bounded.

ii) Define closure of a set. Let A be a non-empty subset of $\mathbb{R}$ and $d(x, A) = inf\{|x-y| : y \in A\}$. Prove that $d(x, A)=0$, if and only if $x \in \overline{A}$, the closure of A.

———